* K Best Improvement Aspiration
  + Another aspect we would have liked to have experimented with is to try a more advanced aspiration criterion. This would allow the penalties to be ignored if a solution existed, such that it was of better quality (lower cost) than the worst of the best Q solutions visited so far. Aspiration moves to the current Q best found solutions would not be allowed, thus ensuring that this scheme will not cause solutions to be revisited. This would allow us to vary the number of aspiration moves. Thus, we would be able to see if allowing more aspiration moves had any effect on the quality of solutions and if it would be possible to produce even better results. Hopefully, this would lead to GLS exploring higher quality basins and plateaus in the search landscape, which might otherwise have been ignored owing to penalties imposed earlier on in the search.
  + We would have like to have experimented with a more advanced aspiration criterion to see if we could improve over the basic improved-best aspiration criterion for GLS. This would allow the penalties to be ignored if a solution existed, such that it was of better quality (lower cost) than the worst of the best Q solutions visited so far. Aspiration moves to the current Q best found solutions would not be allowed, thus ensuring that this scheme will not cause solutions to be revisited. This would allow us to vary the amount of aspiration moves. Thus, we would be able to see if allowing more aspiration moves had any effect on the quality of solutions and if it would be possible to produce even better results. Hopefully, this would lead to GLS exploring higher quality basins and plateaus in the search landscape, which might otherwise have been ignored due to penalties imposed earlier on in the search.
* Varying strength and time of random moves
  + Another possible area for investigation would be to vary when and how random moves are made, in a similar way to Iterated Local Search [88] or Variable Neighbourhood Search [65]. This could be done by restricting when random moves can be made (for example only allowing random moves to be made when a local minimum has been reached) and also varying the "strength" of random moves (how much it changes the solution) by allowing a sequence of random moves to be made, rather than just a single random move. It would have also been interesting to try some other smaller probabilities for executing random moves for GLSQAP, to see if these might work even better than 0.2, as we did in the SAT and MAX-SAT problems, and also experiment with even smaller λ settings for the SAT and MAX- SAT to see if GLS with random moves helps more for these settings. As well as trying different parameter and heuristic combinations, it would also be worth applying GLS with random moves to other problems, particularly those with landscapes similar to the QAP (possibly the Travelling Salesperson Problem), with many local minima basins, but fewer plateaus. This would help verify the results presented in this chapter.
  + We would also have liked to experiment with a more elaborate random move scheme. The first way in which we might improve the scheme would be to only allow random moves to be made once GLS was in a local minimum as an alternative to penalising features in that local minimum. In addition to this, the way in which a random move is made could be extended by allowing a sequence of random moves from the neighbourhood to be made, rather than just one individual random move, thus having a greater effect on the solution. The number of random moves could be fixed according to some parameter setting, be varied randomly, or varied according to the quality of the current local minimum or learnt during the search in some way. By using such a scheme, where larger jumps may be made, this might also alleviate problems with poor random starting points, by allow a partial restarting strategy.
* Ways to improve diversification
  + Having observed how successful the long term memory mechanism of RTS is, we wonder if it would work well with GLS. If we had more time, we would consider replacing the random moves with a similar mechanism. One way in which this might be implemented would be to have an additional set of "negative" penalty terms in the augmented objective function for helping to encourage facility-location pairs (or other types of feature, e.g. edges between cities in the TSP) which had not been present in a solution for a number of repairs to be reintroduced into a solution. These "incentive" penalties would have to be present for a fixed number of iterations after the reintroduction of the feature, to ensure the search had a chance to adapt the current solution to the new solution feature
  + Another aspect is the need to make sure the whole of the search space is explored as evenly as possible for good quality regions. To this end, it may also be useful to look at adding long term memory diversification strategies, such as are used by robust tabu search [89], whereby solution attributes which have not been present for more than a specified number of iterations are forced into the current solution. A softer version of this could be implemented in GLS by the addition of incentives (negative penalties) to the augmented objective function (present before, and then after the desired attribute has been present in solutions for a number fixed number of repairs), which encourage these attributes to be re-introduced into solutions. Because the incentives would not actually force the attributes into solutions, but only "encourage" them with the right amount of incentive, it might be that the local search algorithm would only place these attributes in solutions when they were relatively advantageous to the current area of the search space. This might increase the likelihood of this resulting in higher quality regions of the search space being discovered by such a mechanism.